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COMBINING STATE DEPENDENT RICCATI EQUATION APPROACH WITH DYNAMIC INVERSION: APPLICATION TO CONTROL OF FLIGHT VEHICLES

Rama K. Yedavalli
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State Dependent Algebraic Riccati Equation (SDRE) techniques are rapidly emerging as a design method, which provides a systematic and effective means of designing nonlinear controllers, observers, and filters. This paper describes a new method of integrating the SDRE technique with the Dynamic Inversion control law that is frequently used in the design of aircraft control systems. This paper also provides an example by applying this control design technique to a reusable launch vehicle.

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Combining State Dependent Riccati Equation Approach with Dynamic Inversion: Application to Control of Flight Vehicles

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Abstract

State Dependent Algebraic Riccati Equation (SDRE) techniques are rapidly emerging as a design method, which provides a systematic and effective means of designing nonlinear controllers, observers and filters. This paper describes a new method of integrating the SDRE technique with the Dynamic Inversion control law that is frequently used in the design of aircraft control systems. This paper also provides an example by applying this control design technique to a reusable launch vehicle.

Introduction

There have been a number of design methodologies developed for control of nonlinear systems. The aircraft problem is one such nonlinear system to which control design techniques such as Dynamic Inversion have been applied. Lesser-known nonlinear design procedures are those that involve the state dependent Riccati equations (SDRE). The State Dependent Riccati Equation approach to nonlinear system stabilization relies on representing a nonlinear system's dynamics similar to linear dynamics, but with state dependent coefficient matrices that can be inserted into state dependent Riccati equations to generate a feedback law. Although stability of the resulting closed loop system need not be guaranteed a priori, simulation studies have shown that the method can often lead to suitable control laws.

Over the past several years various SDRE design methodologies have been successfully applied to aerospace problems. SDRE based design procedures have been used in advanced guidance law development [1,2] and in an output feedback autopilot design [3]. Additionally, SDRE design methods have been used in nonlinear filter development [4].

In [5], Ehrler and Vadali investigated the nonlinear regulator problem and showed that solving an algebraic Riccati as it evolved over time provided one means of obtaining a sub optimal solution of the infinite horizon problem. In essence the State Dependent Riccati Equation was treated as being time dependent and its state dependency was not explicitly acknowledged, addressed or analyzed. In [6], SDRE nonlinear regulation, SDRE nonlinear H_∞, and SDRE nonlinear H_2 design methodologies were defined and the optimality, sub optimality and stability properties of SDRE nonlinear regulation was investigated.

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Overview

SDRE stabilization refers to the use of State Dependent Riccati Equations to construct nonlinear feedback control laws for nonlinear systems. The main idea is to represent the nonlinear system

$$\dot{x}=f(x)+B(x)u$$

in the form

$$\dot{x}=A(x).x+\mathbf{B}(x)u$$

and to use the feedback

$$u=-R^{-1}(x)B^{T}(x)P(x)x$$

where P(x) is obtained from the SDRE

$$P(x)A(x)+A^{T}(x)P(x)+Q(x)-P(x)B(x)R^{-1}(x)B^{T}(x)P(x)=0$$

and Q(.) and R(.) are design parameters that satisfy the point wise definiteness condition

$$Q(x)>0 \qquad R(x)>0$$

The resulting closed loop dynamics have a linear-like structure given by

$$\dot{x}=A_{CL}(x).x \ \ where$$
$$A_{CL}(x)=A(x)-R^{-1}(x)B(x)B^{T}(x)P(x)$$

Simulation studies have shown that the dynamics matrix satisfies the Lyapunov Criterion for stability given by

$$P(x)A_{CL}(x)+A_{CL}^{T}(x)P(x)<-Q_{pd} \ \ where$$
$$Q_{pd}>0$$

Equations of Motion of Aircraft

$$\dot{\phi}=P+Q\sin\phi\tan\theta+R\cos\phi\tan\theta$$
$$\dot{\theta}=Q\cos\phi-R\sin\phi$$
$$\dot{\psi}=Q\sin\phi\sec\theta+R\cos\phi\sec\theta$$

$$\dot{P}=c_1RQ+c_2PQ+c_3L+c_4N$$
$$\dot{Q}=c_5PR-c_6(P^2-R^2)+c_7M$$
$$\dot{R}=c_8PQ-c_2RQ+c_4L+c_9N$$

$$\dot{U}=RV-QW+\frac{F_x}{m}-g\sin\theta$$
$$\dot{V}=-RU+PW+\frac{F_y}{m}+g\cos\theta\sin\phi$$
$$\dot{W}=QU-PV+\frac{F_z}{m}+g\cos\theta\cos\phi$$

where

$$c_1=[(J_y-J_z)J_z-J_{xz}^2]/\Gamma$$
$$c_2=(J_x-J_y+J_z)J_{xz}/\Gamma$$
$$c_3=J_z/\Gamma$$
$$c_4=J_{xz}/\Gamma$$
$$c_5=(J_z-J_x)/J_y$$
$$c_6=J_{xz}/J_y$$
$$c_7=1/J_y$$
$$c_8=[(J_x-J_y)J_x+J_{xz}^2]/\Gamma$$
$$c_9=J_x/\Gamma$$

where

$$\Gamma=J_xJ_z-J_{xz}^2$$

State Dependent Linear State Space System

Example 1

State : $x = [P \quad Q \quad R]$

$$\dot{x} = \begin{bmatrix} c_2x_2 & c_1x_3 & 0 \\ -c_6x_1 & 0 & c_5x_1 + c_6x_3 \\ 0 & c_8x_1 & -c_2x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} c_3 & 0 & c_4 \\ 0 & c_7 & 0 \\ c_4 & 0 & c_9 \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Example 2

State : $x = [P \quad Q \quad R \quad \phi \quad \theta \quad \psi]$

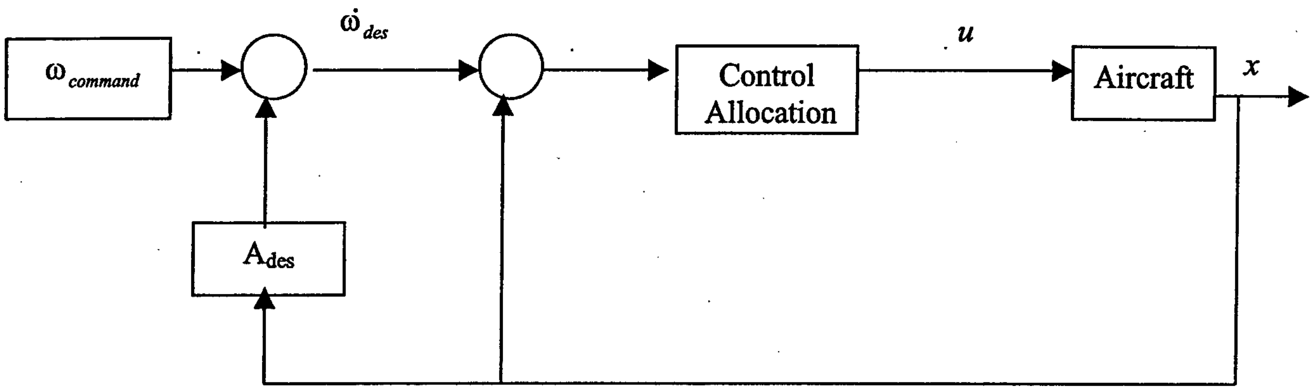
$$\dot{x} = \begin{bmatrix} c_2x_2 & c_1x_3 & 0 & 0 & 0 & 0 \\ -c_6x_1 & 0 & c_5x_1 + c_6x_3 & 0 & 0 & 0 \\ 0 & c_8x_1 & -c_2x_2 & 0 & 0 & 0 \\ 1 & \sin x_5 \tan x_5 & \cos x_5 \tan x_5 & 0 & 0 & 0 \\ 0 & \cos x_4 & -\sin x_4 & 0 & 0 & 0 \\ 0 & \sin x_4 \sec x_5 & \cos x_4 \sec x_5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} c_3 & 0 & c_4 \\ 0 & c_7 & 0 \\ c_4 & 0 & c_9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Example 3

State : $x = [U \quad V \quad W \quad P \quad Q \quad R \quad \sin\theta \quad \cos\theta\sin\phi \quad \cos\theta\cos\phi]$

$$\dot{x} = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4 + c_6x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ F \\ F \\ L \\ M \\ N \end{bmatrix}$$

Schematic Representation of the Method



Combined SDRE and Dynamic Inversion Control Law

Stability of Nominal System for Full State Feedback

$\dot{x} = A(x)x + Bu$

Riccati Based Control Law

$u = -R^{-1}B^{T}P(x)x$

Dynamic Inversion Control Law

$u = B^{-1}(\dot{x}_{des} - A(x)x$

$W.K.T \quad B^{-1} = (B^{T}B)^{-1}$

Let

$\dot{x}_{des} = A_{des}(x)x$

Then

$u = -(B^{T}B)^{-1}B^{T}(A(x) - A_{des}(x))x$

Comparing Equations (1) & (2)

$R = B^{T}B$

$P(x) = A(x) - A_{des}(x)$

The SDARE becomes

$$P(x)A(x) + A^{T}(x)P(x) + Q - P(x)P(x) = 0$$

Solving the above equation for P(x), we can calculate

$$A_{des}(x) = A(x) - P(x)$$

Stability of Nominal System for Output Feedback

$$\dot{x} = A(x)x + Bu$$
$$\omega = Cx$$
$$\dot{\omega} = C\dot{x} = CA(x)x + CBu$$
$$Let \quad \dot{\omega}_{des} = A_{des}(x)x$$

Riccati Based Control Law

$$u = -R^{-1}B^T P(x)x$$

Dynamic Inversion Control Law

$$u = (CB)^+(\dot{\omega}_{des} - CA(x)x)$$
$$u = -(CB)^+(CA(x) - A_{des}(x))x$$
$$u = -\{(CB)^T(CB)\}^{-1}(CB)^T(CA(x) - A_{des}(x))x$$
$$u = -(B^T C^T CB)^{-1} B^T C^T (CA(x) - A_{des}(x))x$$

Comparing Equations (3) & (4)

$$R = B^T C^T CB$$
$$P(x) = C^T(CA(x) - A_{des}(x))$$

The SDARE becomes

$$P(x)A(x) + A^T(x)P(x) + Q - P(x)B(B^T C^T CB)^{-1} B^T P(x) = 0$$

Solving the above equation for P(x), we can calculate A_{des} using the equation

$$P(x) = C^T(CA(x) - A_{des}(x))$$

Closed Loop System

$$\dot{x} = [A(x) + B(CB)^+(A_{des}(x) - A(x))]x = A_c(x)x$$

Verification of Stability

Let P_L be a positive definite matrix, which is chosen as $P_L = P(x_0)$. $P(x_0)$ is the solution to the SDARE at the initial condition $\{x_0\}$.

Full State Feedback

$$P(x_0)A(x_0)+A^T(x_0)P(x_0)+Q-P(x_0)P(x_0)=0$$

Output Feedback

$$P(x_0)A(x_0)+A^T(x_0)P(x_0)+Q-P(x_0)B(B^TC^TCB)^{-1}B^TP(x_0)=0$$

The Closed Loop System is locally asymptotically stable if

$$P_LA_c(x)+A_c^T(x)P_L<0$$

Application of Combined SDRE and Dynamic Inversion Control Law to Aircraft

$$State\ :\ x=[U\ \ V\ \ W\ \ P\ \ Q\ \ R\ \ \sin\theta\ \ \cos\theta\sin\phi\ \ \cos\theta\cos\phi]$$

$$\dot{x}=\begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4+c_6x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}+\begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} F_x \\ F_y \\ F_z \\ L \\ M \\ N \end{bmatrix}$$

where

$$A(x)=\begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4+c_6x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \& B=\begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above state space system is not completely controllable. However if we separate the A(x) matrix into $A_1(x)$ and $A_2(x)$ given by

$$A_1(x)=\begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4+c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad and \quad A_2(x)=\begin{bmatrix} 0 & -x_6 & x_5 \\ x_6 & 0 & x_4 \\ -x_5 & -x_4 & 0 \end{bmatrix}.$$

then the pair ($A_1(x)$,B_1) are completely controllable.

$$A_1(x)=\begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4+c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad and \quad B_1=\begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \end{bmatrix}$$

Therefore we can design a control law given by

$$u = B_1^{-1}(\dot{x}_{des} - A_1(x)x_1$$
$$where \quad x_1 = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]$$
$$and \quad u = [F_x \quad F_y \quad F_z \quad L \quad M \quad N]$$
$$W.K.T \quad B_1^{-1} = (B_1^T B_1)^{-1}$$
Let
$$\dot{x}_{1des} = A_{1des}(x)x_1$$
Then
$$u = -(B_1^T B_1)^{-1} B_1^T (A_1(x) - A_{1des}(x))x_1$$

$A_{1des}(x)$ is calculated from the equation

$$A_{1des}(x) = A_1(x) - P(x)$$

where P(x) is the solution to the Riccati Equation

$$P(x)A_1(x) + A_1^T(x)P(x) + Q - P(x)B_1(x)R^{-1}(x)B_1^T(x)P(x) = 0$$

However we know that

$$R = B_1^T B_1$$

Therefore the State Dependent Algebraic Riccati Equation reduces to

$$P(x)A_1(x)+A_1^T(x)P(x)+Q-P(x)P(x)=0$$

The closed loop system is given by

$$\dot{x}_1 = A_1(x)x + B_1[B_1^{-1}(A_{1des}(x)-A_1(x))x_1]$$

Therefore, we have

$$\dot{x}_1 = A_{C1}(x)x_1$$
$$where\ \ A_{C1}(x)=A_1(x)+B_1[B_1^{-1}(A_{1des}(x)-A_1(x))]$$

Verification of Closed Loop System Stability

In the previous section we separated A(x) into

$$A_1(x)=\begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4+c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad and\ \ A_2(x)=\begin{bmatrix} 0 & -x_6 & x_5 \\ x_6 & 0 & x_4 \\ -x_5 & -x_4 & 0 \end{bmatrix}$$

The matrix $A_2(x)$ is neutrally stable if we calculate it's eigenvalues by freezing the states at each time instant. However we are more concerned about the stability of matrix $A_1(x)$ under the control 'u' that we previously discussed. The closed loop system under control 'u' is given by $A_{c1}(x)$.
Let P_L be a positive definite matrix, which is chosen as $P_L = P(x_{10})$. $P(x_{10})$ is the solution to the SDARE at the initial condition $\{x_{10}\}$.

$$P(x_{10})A(x_{10})+A^T(x_{10})P(x_{10})+Q-P(x_{10})P(x_{10})=0$$

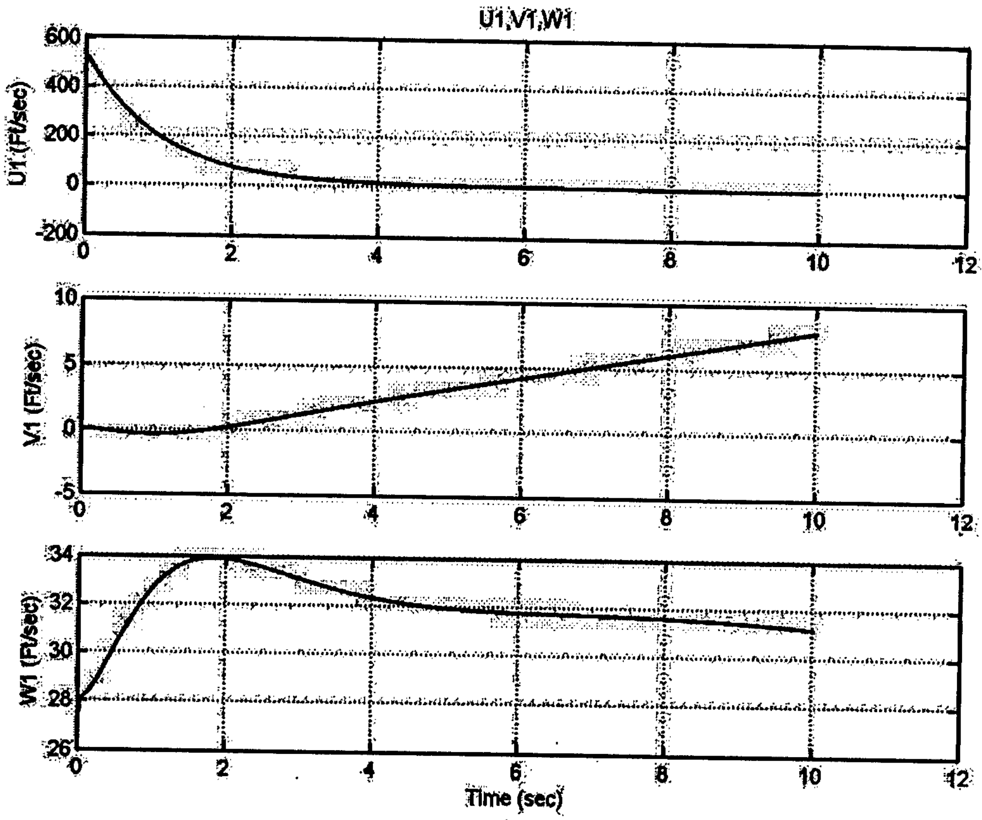
The closed loop system is locally asymptotically stable if

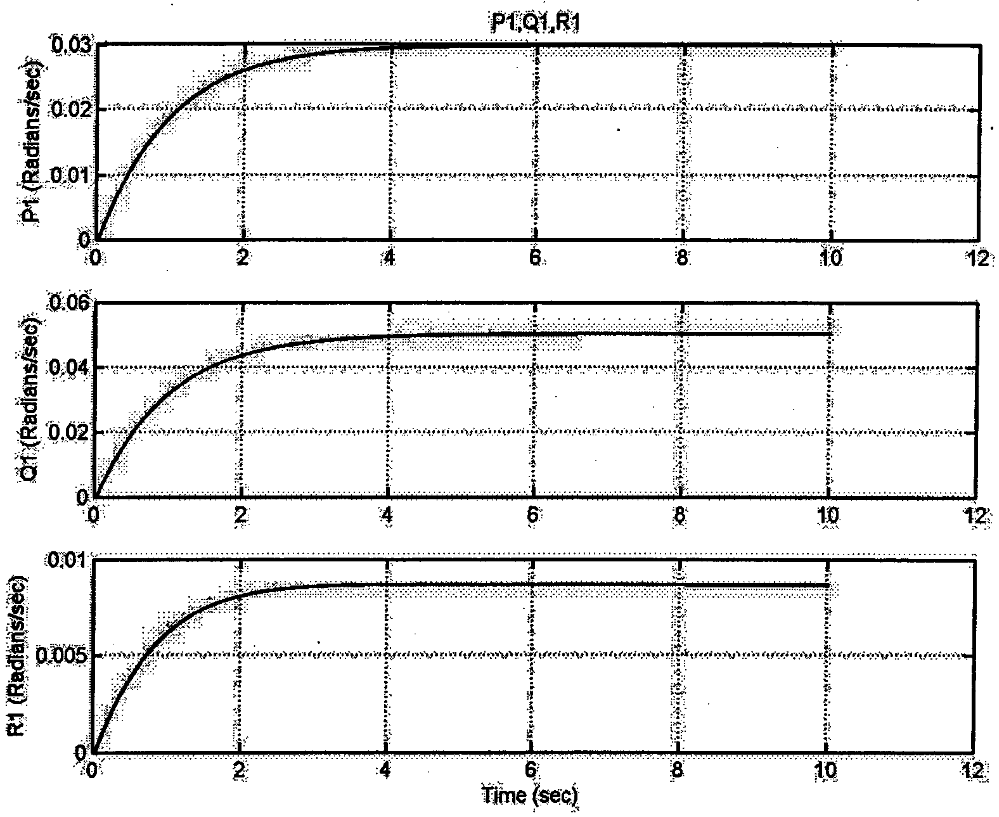
$$P_LA_{c1}(x)+A_{c1}^T(x)P_L<0$$

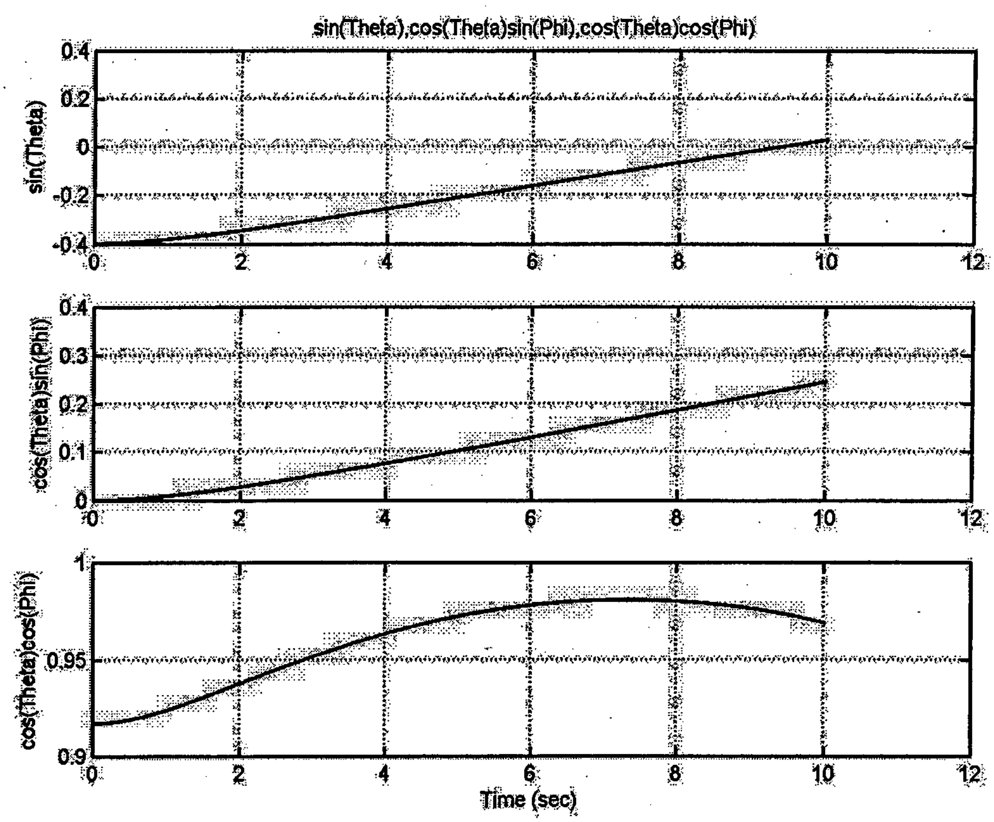
Results

Initial Condition 1
Uo = 517.5

Vo = 0
Wo = 27.5
Po = 0
Qo = 0
Ro = 0




P1,Q1,R1
P1 (Radians/sec)
Q1 (Radians/sec)
R1 (Radians/sec)
Time (sec)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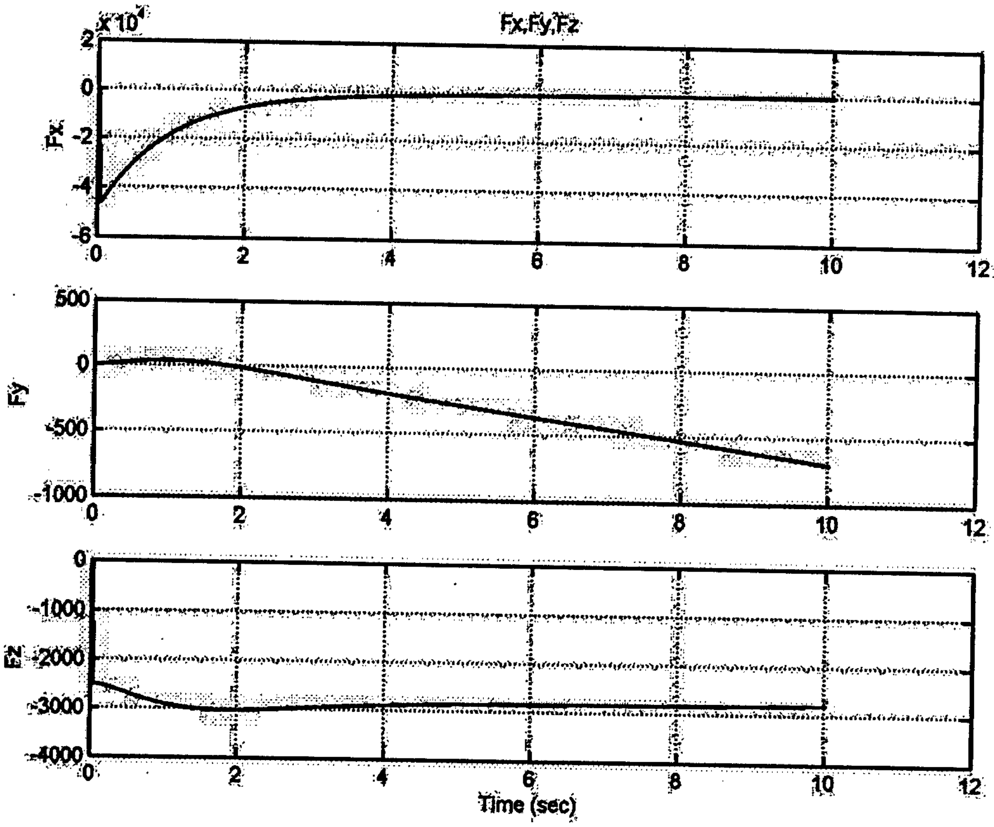
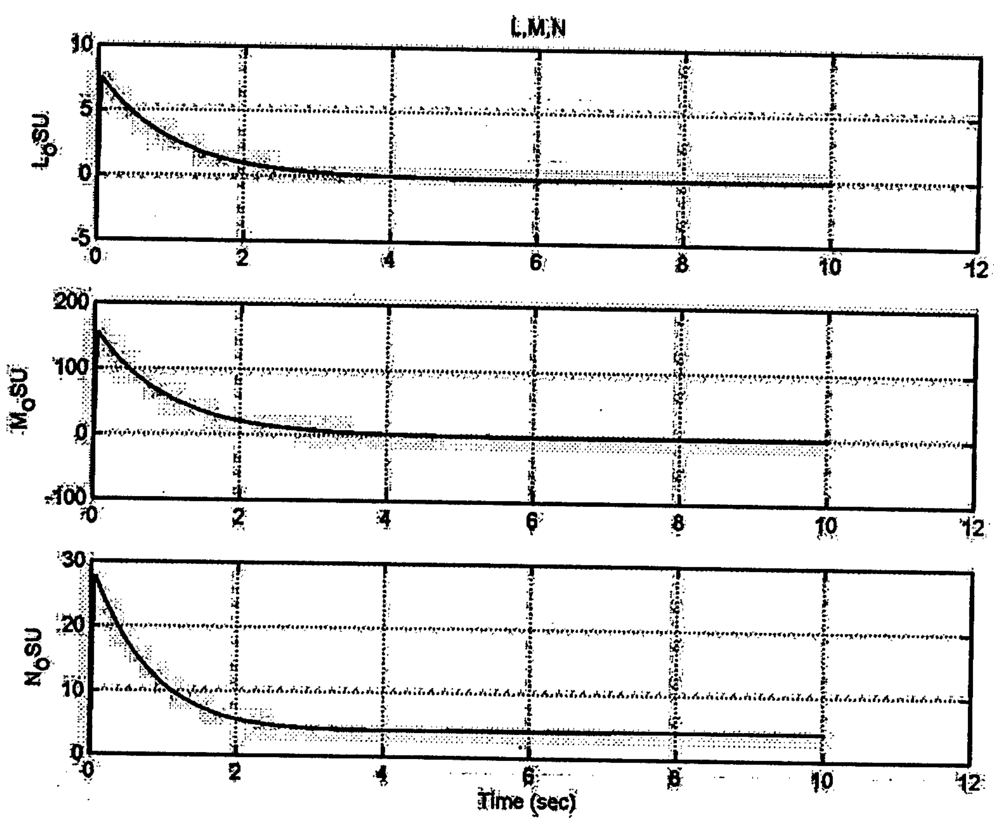
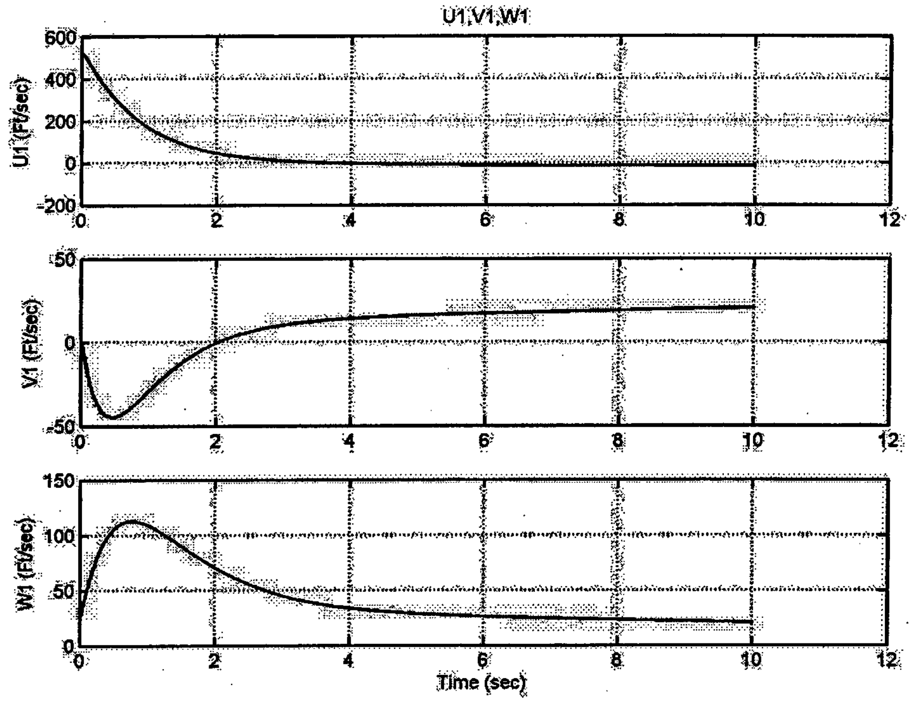
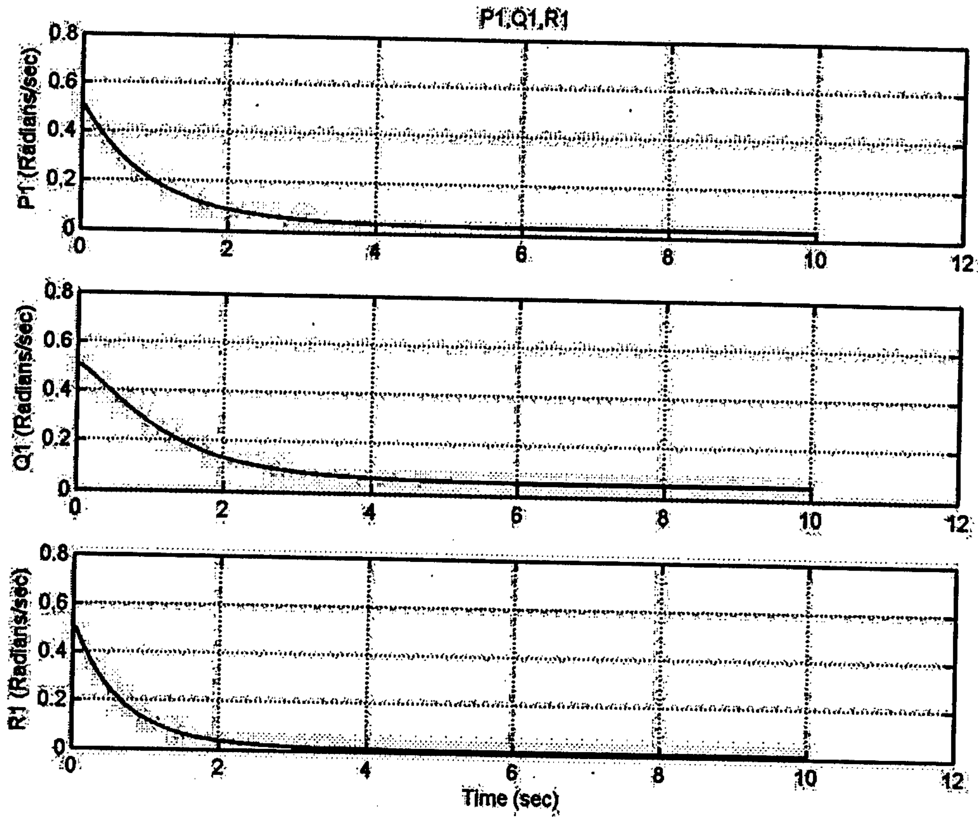
sin(Theta)
cos(Theta)sin(Phi)
cos(Theta)cos(Phi)
Time (sec)

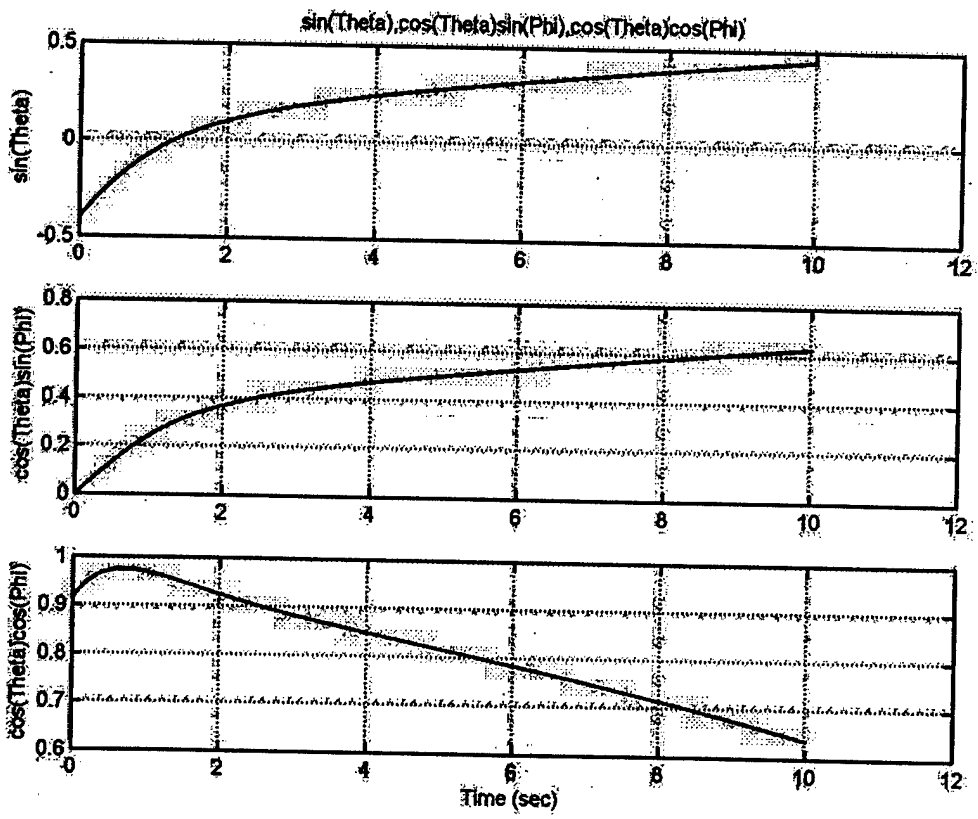

Fx,Fy,Fz
x 10^4
Fx
Fy
Fz
Time (sec)

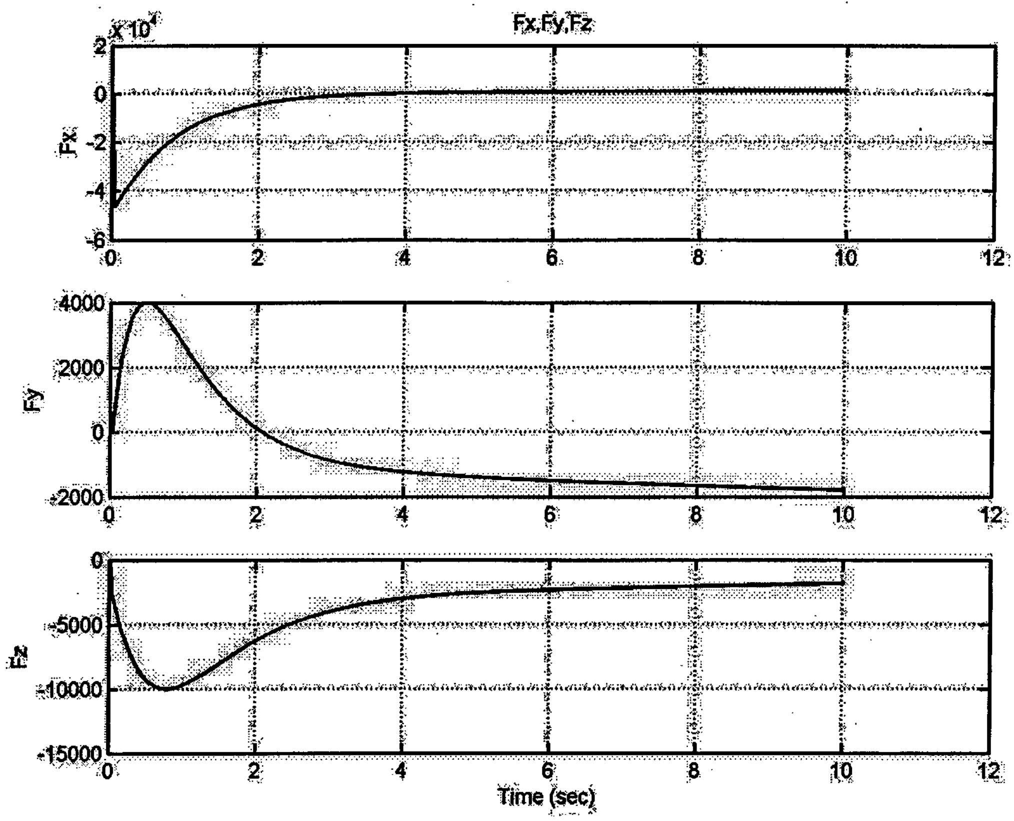

L,M,N
L_OSU
M_OSU
N_OSU
Time (sec)

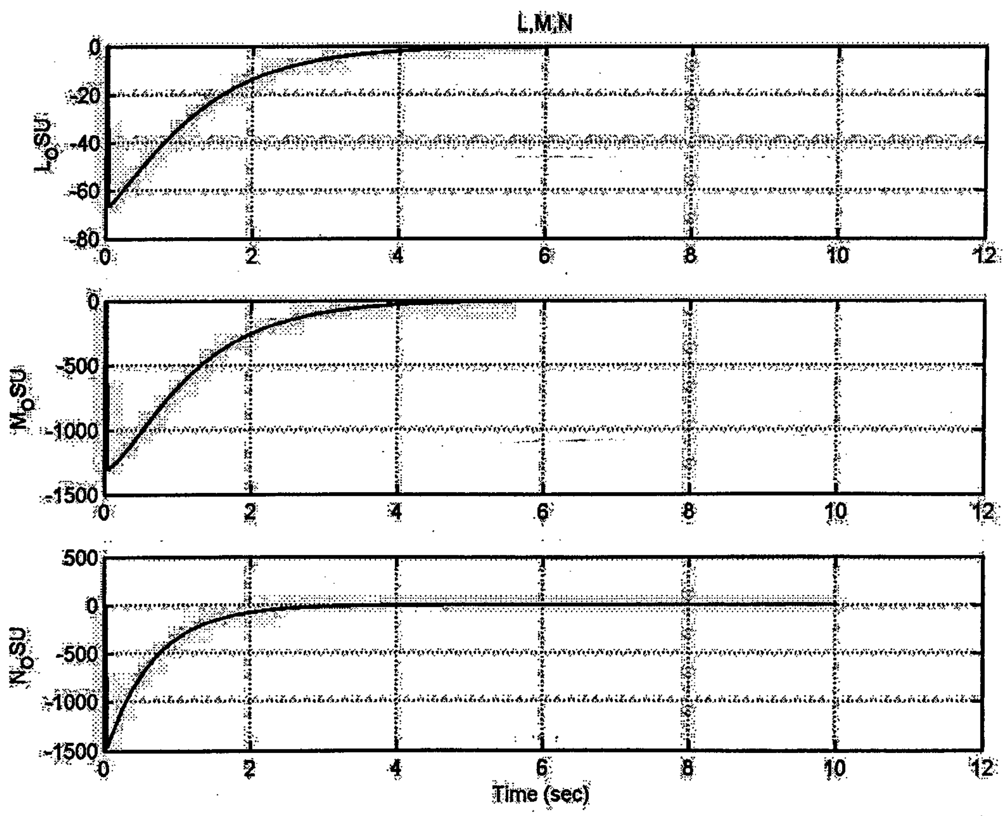
Results
Initial Condition 2
Uo = 517.5
Vo = 0
Wo = 27.5
Po = 0.5
Qo = 0.5
Ro = 0.5




P1,Q1,R1
P1 (Radians/sec)
Q1 (Radians/sec)
R1 (Radians/sec)
Time (sec)


sin(Theta),cos(Theta)sin(Phi),cos(Theta)cos(Phi)
sin(Theta)
cos(Theta)sin(Phi)
cos(Theta)cos(Phi)
Time (sec)


Fx,Fy,Fz
x 10^4
Fx
Fy
Fz
Time (sec)


L,M,N
L_oSU
M_oSU
N_oSU
Time (sec)

Conclusions

In this paper a new control law was developed that is a combination of the existing State Dependent Riccati Equation techniques and the Dynamic Inversion control law. This control system design was then applied to the aircraft dynamics and the resulting closed loop system was shown to be stable even under change in the initial conditions.

References

[1] J.R Cloutier *"Adaptive matched augmented proportional navigation."* Presented at AIAA Missile Sciences Conference, November 1994.

[2] J.R. Cloutier *"Time-to-go-less guidance with cross-channel couplings."* Proceedings of the AIAA Missile sciences Conference, Monterey, CA, December 1996.

[3] C.P. Mracek and J.R. Cloutier *"Missile longitudinal autopilot design using the state dependent Riccati equation method."* Proceedings of the International Conference on Nonlinear Problems in Aviation and Aerospace, May 1996.

[4] C.P Mracek, J.R Cloutier, and C.N. D'Souza *"A new technique for nonlinear estimation."* Proceedings of the IEEE Conference on Control Applications, Dearborn, MI, September 1996.

[5] D. Ehrler and S.R. Vadali *"Examination of the optimal nonlinear regulator problem."* Proceedings of the AIAA Guidance, Navigation and Control Conference, Minneapolis, MN, August 1988.

[6] J.R. Cloutier, C.N. D'Souza, and C.P. Mracek *"Nonlinear regulation and nonlinear H_∞ control via the state dependent Riccati equation technique; part 1, theory; part 2, examples."* Proceedings of the International Conference on Nonlinear Problems in Aviation and Aerospace, May 1996.

[7] J.R. Cloutier *"State Dependent Riccati Equation Techniques: An Overview"* Proceedings of American Control Council Conference, June 1997

[8 Zhihua Qu and J.R. Cloutier *"A New Sub-optimal Control for Cascaded Nonlinear Systems"* Proceedings of American Control Council Conference, June 2000

[9] J.R Cloutier and D.T. *Stansbery "The Capabilities and Art of State-Dependent Riccati Equation-Based Design"* Proceedings of American Control Council Conference, June 2002

[10] *"Dynamics of Flight Stability and Control"* Bernard Etkin and Lloyd Duff Reid, John Wiley and Sons, 3rd Ed.

[11] *"Aircraft Control and Simulation"* Brian L. Stevens and Frank L. Lewis, John Wiley and Sons

[12] *"Robust Control of Nonlinear Uncertain Systems"* Zhihua Qu, John Wiley and Sons

[13] D.B Doman and A.D Ngo *"Dynamic Inversion Based Adaptive/Reconfigurable Control of the X-33 on Ascent"* Journal of Guidance Control and Dynamics, 2002